Complex dynamics on a Monopoly Market with Discrete Choices and Network Externality

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Abstract

In this paper, we explore the effects of the introduction of localised externalities through interaction structures upon local and global properties of the simplest market model: the discrete choice model with a single homogeneous product and a single seller (monopoly). Following Kirman, the resulting market is viewed as a complex interactive system. We use an ACE (Agent based Computational Economics) approach to investigate the market mechanisms and underline in what way the knowledge of generic properties of complex adaptive system dynamics can enhance our perception of the market mechanism in the numerous cases where individual decisions are inter-related.

Résumé *Dans ce papier, nous explorons les effets de l'introduction d'externalités transmises par des structures d'interactions sur les propriétés locales et globales du plus simple modèle de marché : choix discret, produit homogène, monopole. A la suite de Kirman, le marché ainsi décrit peut être considéré comme un système complexe interactif. Nous utilisons une approche multi-agents (ACE) pour explorer les mécanismes de marché et monter de quelle manière les propriétés génériques des systèmes complexes adaptatifs peuvent améliorer notre perception des phénomènes de marchés lorsque les choix des agents sont interdépendants.*

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1 Introduction

In this paper, we explore the effects of the introduction of interaction structures (structured externalities) upon local and global properties of the simplest market model: the discrete choice model (Anderson *et al.* [1]) with a single homogeneous product and a single seller (the monopoly case). Following Kirman [12, 13, 14] such a market is viewed as a *complex interactive system* with a communication network. First part of the present study relies on numerical simulations, making use of 'Moduleco', a multi-agent plateform [25, 17, 18].

2 local and global interaction on a network

In this section, based on Phan,Pajot,Nadal [20] [1], we focus on the effect on market behaviour of interdependencies between customers, and study an economic model which allows us to highlight the economic effects of the structure interdependencies on the global level of demand. Three situations are possible (Table 1).

2.1 Individual interactions and the demand : a typology of neighbourhood

In the first extreme case (a), there are no relations between agents (Moduleco: "empty" neighbourhood). In this case, the aggregate demand is independent of the structure and no external effect (local or global) is present. The agents are independent one from each other.

In the second extreme case (c), all agents are connected by direct relations in the system (Moduleco: "world" neighbourhood). All agents are equivalent in the network and they interact by means of global interactions. In this way, the aggregate demand is sensitive to the global external effect initiated by the sensitivity of agents to the choices of others, but remains independent of the topology of the network (because the neighbourhood of each agent is composed of all the other agents). Thus, finite sequences of interdependent buy decisions called "avalanches" arise, but such "dominoes effect" are not localised on the network but appear in a dispersed way within the system.

Finally, the intermediate case (b) corresponds to situations where agents have specified relations (regular neighbourhood or not). Interactions between agents are local, and the topology of the interpersonal network influences the aggregated demand on the market. This local interdependence gives rise to localised avalanches on the network.

Table 1: A typology of interactions and demand dynamics

Neighbourhood *(Moduleco*)*	(a) No relations *(empty)*	(b) Localised relations *(neighb2, neighb4,...)*	(c) Generalised relations *(world)*
Level of interactions	(independent agents)	Localized interactions	Global interactions
Demand sensitivity to the network topology	Null	Strong	Null
Avalanches	No	Possible (localised in the network)	Possible -not localised in the network)

2.2 Modelling the individual choice in a social context

This model deals with the simplest discrete choice problem (Anderson *et al.*, [1]): binary choice. We focuse more specifically on dynamic aspects, including effects depending on the network architecture.

We consider a set Ω_N of N agents with a classical linear willingness-to-pay function. Each agent $i \in \Omega_N$ either buys ($\omega_i = 1$) or does not buy ($\omega_i = 0$) one unit of the single given good of the market. A rational agent chooses ω_i in order to maximize his *surplus function* V_i:

[1] available at: http://www-eco.enst-bretagne.fr/~phan/papers/ppn2003.pdf

$$\max_{\omega_i \in \{0,1\}} V_i = \max_{\omega_i \in \{0,1\}} \omega_i (H_i + \sum_{k \in \vartheta_i} J_{ik}\omega_k - P), \tag{1}$$

where P is the price of one unit and H_i represents the idiosyncratic preference component. Some other agents k, within a subset $\vartheta_i \subset \Omega_N$, such that $k \in \vartheta_i$, hereafter called neighbours of i, influence agent i's preferences through their own choices ω_k. This social influence is represented here by a weighted sum of these choices. Let us denote J_{ik} the corresponding weight i.e. the marginal social influence on agent i, of the decision of agent $k \in \vartheta_i$. When this social influence is assumed to be positive ($J_{ik} > 0$), it is possible, following Durlauf [8, 9], to identify this external effect as a *strategic complementarity* in agents' choices [6, 7].

Formally, if we define the social influence component by a continuous $\mathcal{C}^2$ function: $S(\omega_i, \omega_{-i})$ where ω_{-i} is the vector of the neighbours' choices :

$$S(\omega_i, \omega_{-i}) \equiv \omega_i \sum_{k \in \vartheta_i} J_{ik}\omega_k \tag{2}$$

the social influence component in specification (1) appears to be a restriction of (2) at binary arguments $\{0, 1\}$. In the continuous case, the marginal social parameter J_{ik} appears to be the second order cross-derivative of $S(\omega_i, \omega_{-i})$ with respect to ω_i and ω_k, according to the definition of strategic complementarity:

$$\frac{\partial^2 S(\omega_i, \omega_{-i})}{\partial \omega_i \partial \omega_k} = J_{ik} > 0 \tag{3}$$

For simplicity, we consider here only the case of *homogeneous* influences, that is, identical positive weights $J_{ik} = J_{\vartheta_i}$ for all influence parameters in the neighbourhood of i. That is, if N_{ϑ_i} denotes the number of neighbours of agent i, we have :

$$J_{ik} = J_{\vartheta_i} \equiv J/N_{\vartheta_i} > 0 \qquad \forall i, k \in \Omega_N \tag{4}$$

For a given neighbour k taken in the neighbourhood ($k \in \vartheta$), the social influence is J_{ϑ_i} if the neighbour is a customer ($\omega_k = 1$), and zero otherwise. Individual influence is inversely proportional to the size of the neighbourhood. As the cumulated social effect is the sum of individual effects over the neighbourhood, social influence depends on the proportion of customers in the neighbourhood. In a *regular* network (N_{ϑ_i} constant and equal to N_ϑ for all $i \in \Omega_N$), all individual effects have the same magnitude over the network (equal to: $J_\vartheta \equiv J/N_\vartheta$).

Depending on the nature of the idiosyncratic term H_i, the discrete choice model (1) may represent two quite different situations [16, 19]. In this paper, each agent is assumed to has a willingness to pay that is *invariable* in time, but may differ from one agent to the other. As consequence, private idiosyncratic terms H_i are randomly distributed among the agents at the beginning, but remain fixed during the period under consideration. It is useful to introduce the following notation:

$$H_i = H + \theta_i, \tag{5}$$

and to assume that the θ_i are logistically distributed with zero mean and variance $\sigma^2 = \pi^2/(3\beta^2)$ over the population. This assumption implies:

$$\lim_{N\to\infty} \frac{1}{N}\sum_i \theta_i = 0 \quad \text{and} \quad \lim_{N\to\infty} \frac{1}{N}\sum_i H_i = H \tag{6}$$

For a given distribution of choices in the neighbourhood and for a given price, the customer's behaviour is deterministic. An agent buys if :

$$\theta_i > P - H - J_\vartheta \sum_{k\in\vartheta_i} \omega_k, \tag{7}$$

In the full connectivity case (global externality), it is convenient to identify a *marginal customer*, indifferent between buying and not buying. Let $H_m = H + \theta_m$ be his idiosyncratic willingness to pay. This *marginal customer* has zero surplus ($V_m = 0$), that is:

$$\theta_m = P - H - \frac{J}{N-1}\sum_{k\in\vartheta} \omega_k. \tag{8}$$

In this case, an agent buys if: $\theta_i > \theta_m$ and does not buy otherwise.

3 Aggregate demand and collective dynamics.

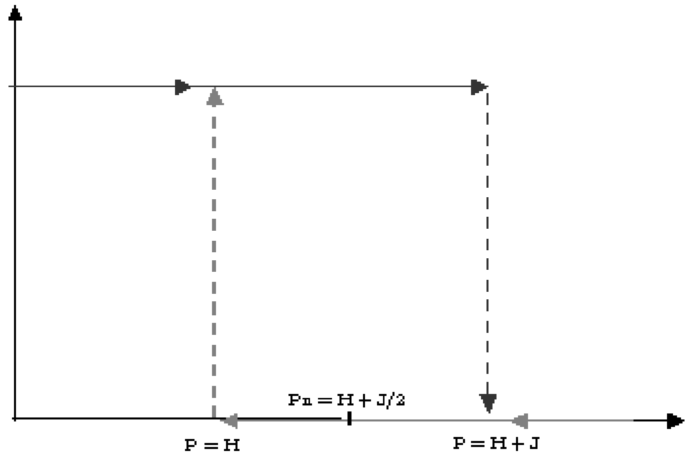
In this class of models, the adoption by a single agent in the population (a "direct adopter") may lead to a significant change in the whole population through a chain reaction of "indirect adopters" As a result the aggregate demand dynamics present singular behaviour at the collective level, according to those observed in the *Random Field Ising Model* studied in statistical physics [16, 19]. This section first reviews such dynamic results obtained by the way of simulations and then provides some analytical features in the special case of "global" externality, which corresponds to the "*mean field*" approximation in statistical physics.

3.1 Avalanches and hysteresis loops in aggregate demand

In the presence of externality, two different situations - or "phases" - may exist, depending on the price: one with a small fraction of adopters and one with a large fraction. By varying the price, a *transition* may be observed between these phases. The jump in the number of buyers occurs at different price values according to whether the price increases or decreases (*hysteresis*), leading to *hysteresis loops* as presented below.

If the external fields were *uniform*, $H_i = H$, for all i, the model would be equivalent to the classic *Ising* model in an external field: $H - P$. In such a case, one would have a first order transition, with all the population abruptly adopting as H passes through zero from below (and vice versa). In figure 1, the initial (decreasing) price threshold is: $P = H$, where the whole population abruptly adopts. After adoption, the (decreasing) price threshold is: $P = H + J$, where the whole population abruptly leaves the market. When all customers are adopters, price variations between $P = H$ and $P = H + J$ have no effect on demand.

Figure 1: Hysteresis with uniform idiosyncratic willingness to pay



In the presence of *quenched disorder* (non uniform H_i), the number of customers evolves by a series of cluster flips, or avalanches. If the disorder is strong enough (the variance σ^2 of H_i is large - or β is small - compared to the strength of the coupling J), there will be only small avalanches (each agent following his own H_i). If σ^2 is small enough (β large), the phase transition occurs through a unique "infinite" avalanche, like in the uniform case. In intermediate regimes, a distribution of avalanches of all sizes can be observed.

From the theoretical point of view, there is a singular price P_n, which corresponds to the *unbiased* situation, that is, the situation where the willingness to pay is neutral on average: there are as many agents likely to buy as not to buy ($\eta = 1/2$).

Suppose that we start with a network in such a neutral state. Then, on average, the willingness to pay of any agent i is $H_i + J/2 - P$, its average over a set of agents great enough being: $H + J/2 - P$. Thus, the neutral state is obtained for

$$P_n = H + J/2. \tag{9}$$

For $P < P_n$, there is a net bias in favour of " buy" decisions ($H + J/2 - P > 0$), whereas for $P < P_n$, there is a net bias in disfavour of " buy" decisions. The main question for $P = P_n$, is to know whether, in this a priori neutral (unbiased) situation,

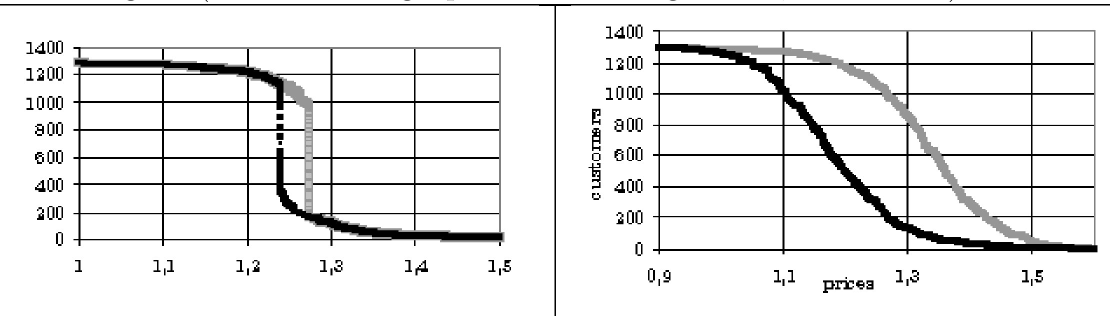
- either this symmetry will reveal itself in the dynamics: starting from, say, a majority of " buy" decisions, the dynamics will drive the system towards a symmetric state, with as many buyers as non buyers (where essentially every agent follows his own bias, $\omega_i = 1$ if $H_i - p > 0$);

- or if there is symmetry breaking, where, e.g. a majority of agent will buy even if one starts with an initial state with as many buyers as non buyers.

One result is that, in the " mean field" analysis (valid for long range interactions - or full connectivity), for a symmetric distribution of the centred idiosyncratic willingness to pay θ_i, one will necessarily observe the first situation if the distribution of the θ_i, has a *maximum* at $\theta_c = 0$, ($H_c = H$), and the second situation may be observed for distributions showing a *minimum* at $\theta_c = 0$. At P not equal to P_n, it is the hysteresis phenomenon which will be the most interesting

It is useful to consider a simple example of a simulation, using the multi-agent framework Moduleco [25, 17, 18]. For the simulations presented below, we have $H = 1$ and $J = 0.5$. For a given variation in price, it is possible to observe the resulting variation in demand. The most spectacular result arises in the case of global interactions (complete connectivity) when nearly all agents update their choices simultaneously (synchronous activation regime,Modumeco : "world").

Figure 2: Hysteresis in the trade-off between prices and customers under synchronous activation regime (Moduleco: Logit pseudo-random genrator, seed = 190)



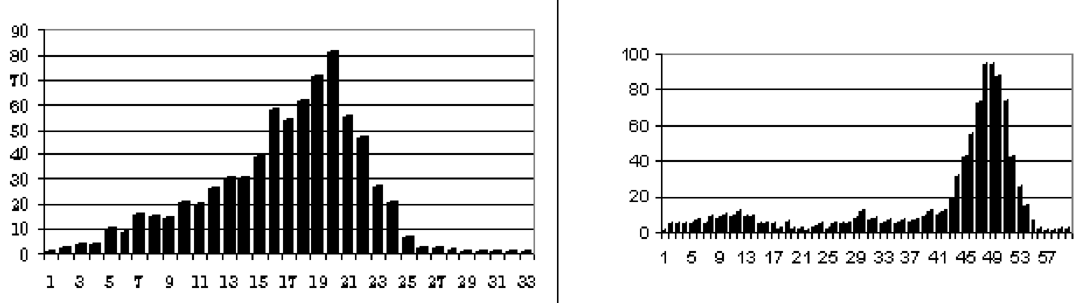
a - global externality, neigh. = world

b - circle, neighbours = 2

parameters: $H = 1$, $J = 0.5$, $\beta = 10$ - upstream (black)and downstream (grey) trajectories

Figure(s) 2 shows the set of the aggregate consumer'sequilibrium positions for the whole demand system over all prices, incremented in steps of 10^{-4}, within the interval [0.9, 1.6] under the synchronous activation regime. The relevant parameters are: $H = 1$, $J = 0.5$, $\beta = 10$. One observes a hysteresis phenomenon with phase transitions around the theoretical point of symmetry breaking: $P_n = H + J/2 = 1.25$. Figure 2a shows the details of straight hysteresis corresponding to the "global" externality (complete connectivity). In this case, the trajectory is no longer gradual, like in the local interdependence case (Figure 2b-d). A succession of growing avalanches arises for $P = 1.2408$, driving the system from an adoption rate of 30% towards an adoption rate of roughly 87%, along the upstream equilibrium trajectory (with decreasing prices). Along the downstream trajectory (with increasing prices) the externality effect induces a strong resistance of the demand system against a decrease in the number of customers. The phase transition threshold is here around $P = 1.2744$. At this threshold, the equilibrium adoption rate decreases dramatically from 73% to 12,7%. Figures 2(b) deal with global' externality, while Figures 2(b) corresponds to a "local" externality (on a one-dimensional periodic lattice: the circle case, with 2 nearest neighbours) with the same parameters in both cases.

Figure 3: Chronology and sizes of induced adoptions in the avalanche at the phase transition under global externality



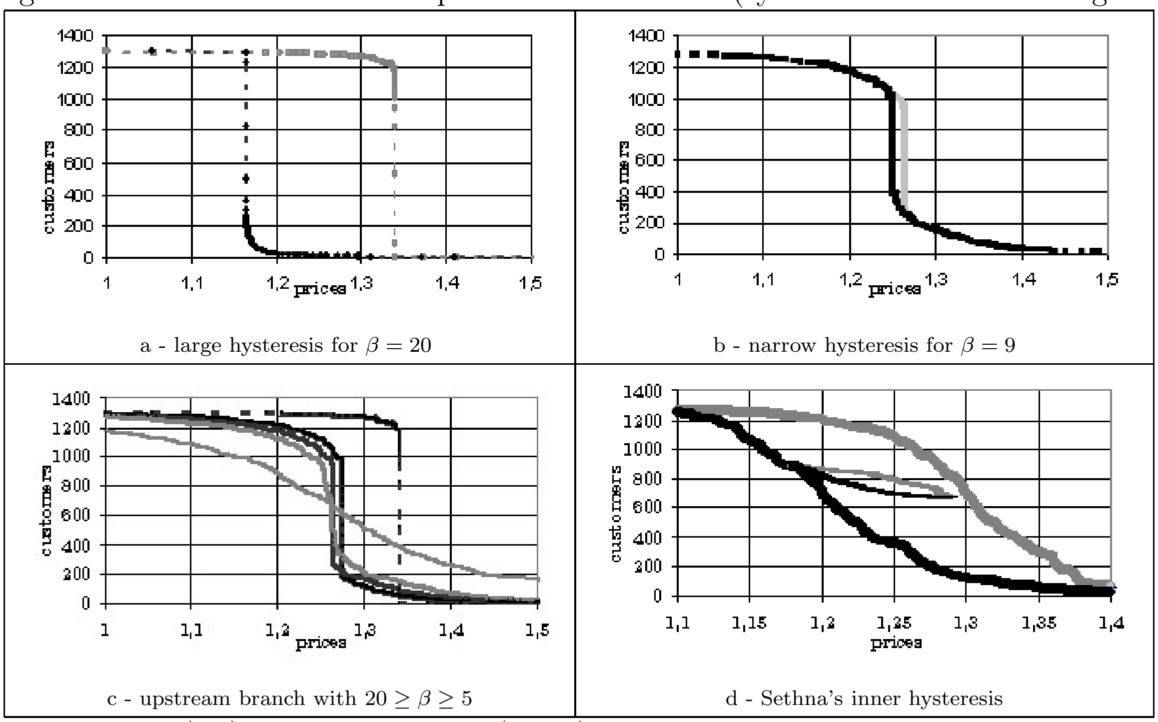
a - the avalanche for p = 1.2408 (seed = 190)

b - avalanche for p = 1.2415 (seed = 40)

parameters: $H = 1$, $J = 0.5$, $\beta = 10$ Moduleco: synchronous activation regime).

Figure 3a shows the chronology of avalanches in the case of the upstream branch of the equilibrium trajectory, for $P = 1.2407$. The evolution follows a smooth path, with a first period of 19 steps, where the initial change of one customer leads to growing avalanches from size 2 to size 81 (6,25% of the whole population). After this maximum, induced changes decrease in 13 steps, including 5 of size one only. Figure 3b shows a different case, with more important avalanches, both in size and in duration (seed 40). The initial impulsion is from a single change for $P = 1.2415$ with a rate of adoption of 19,75%. The first wave includes the first 22 steps, where induced changes increase up to a maximum of 11 and decrease towards a single change. During this first sub-period, 124 people change (9,6% of the whole population). After step 22, a new wave arises with a growing size in change towards a maximum of 94 agents both in periods 48 and 49. The total avalanche duration is 60 steps, where 924 induced agent changes arise (71% of the population - 800 in the second wave).

Figure 4: The trade-off between prices and customers (synchronous activation regime)



a - large hysteresis for $\beta = 20$

b - narrow hysteresis for $\beta = 9$

c - upstream branch with $20 \geq \beta \geq 5$

d - Sethna's inner hysteresis

Fig. (a-c): total connectivity (world) ; d circle with neighbourhood = 8

As suggested previously, the steepness of the phase transition increases when the variance $\sigma^2 = \pi^2/(3\,\beta^2)$ of the logistic distribution decreases (that is, increasing β). The closer the preferences of the agents to each other, the greater is the size of avalanches at the phase transition (Figure 4a-b). Figure 4c shows a set of upstream trajectories for different values of β taken between 20 and 5 in the case of global externality. The scope of the hysteresis decrease with β ; for $\beta < 5$ there is no longer any hysteresis at all. Finally, following results for Random Field Ising Model by Sethna [22],inner sub-trajectory hysteresis can be observed in this case (Figure 4d). Here, starting from a point on the upstream trajectory, an increase in price induces a less than proportional decrease in the number of customers (grey curve). The return to the exact point of departure when the prices decrease back to the initial value (black curve) is an interesting property of Sethna's inner hysteresis phenomenon [20]

3.2 Demand function for the global externality case (mean field): analytic issues

In this subsection we restrict our investigation to the "global" externality case with homogeneous interactions and full connectivity, which is equivalent to the *mean field theory* in physics. Consider the penetration rate η, defined as the fraction of agents that choose to buy at the given price, (i.e. those with $\theta_i > \theta_m$ in 8). In the large N limit, we have $\sum_{k \in \vartheta} \omega_k/(N-1) \approx \eta$, so that: $\theta_m \approx z(\eta)$, where :

$$z(\eta) \equiv P - H - J\,\eta. \tag{10}$$

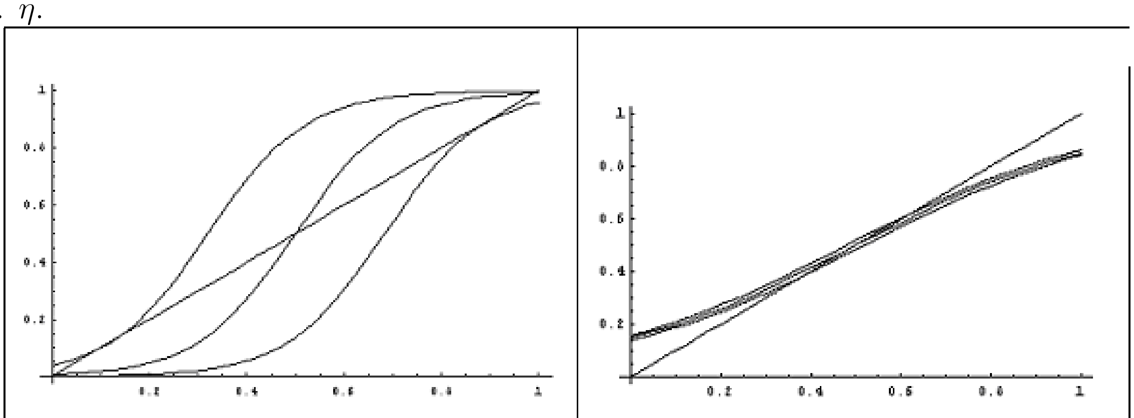
This approximation of (8) allows us to define η as a fixed point:

$$\eta = 1 - F(z(\eta)) \tag{11}$$

where z depends on P, H, and η. Using the logistic distribution for θ_i, we have :

$$\eta = \frac{1}{1 + \exp\left(+\beta\, z\right)} \tag{12}$$

Figure 5: Fixed points for the penetration rate (market demand for a given price): $1-F(z)$ vs. η.



a - equation (12) for $\beta = 20$ | b - equation (12) for $\beta = 6$

As just observed with the hysteresis loop in the previous subsection, for given J, H and P, a multiple fixed point for η may appear for high values of β (low value of σ). In figure 5a, for $\beta = 20$, we can observe the two zones, which are included, roughly speaking, between $P = 1.15$ and $P = 1.35$. $P_n = 1.25$ is the unbiased price. Within this zone, we have two stables fixed point for a given price (and one unstable solution), respectively : $\eta_+(P) > 0.5 > \eta_-(P)$(theses fixed point correspond respectively to the high penetration rate and the low one for a given price, within the hysteresis loop). For a price lower than 1.15 (or a price higher than 1.35), we have a single fixed point (corresponding to the single branch demand curve outside of the loop). In figure 5b, $\beta = 6$, we have a single fixed point for all values of P, corresponding to a single consumer's equibrium of the demand for each price, and a related aggregate "classic" bijective relationship for the demand curve.

Equation (11) allows us to define the penetration rate (an index of the global demand *in proportion*, without any dimension) as an implicit function of the price

$$\Phi(\eta, P) \equiv \eta + F(P - H - J\,\eta) = 1 \tag{13}$$

$$\eta^d(P) + F(P - H - J\,\eta^d((P)) = 1 \tag{14}$$

The shape of this (implicit) demand curve $\eta^d(P)$ can be evaluated using the implicit derivative theorem:

$$\frac{d\eta^d(P)}{dP} = \frac{-\partial\Phi/\partial P}{\partial\Phi/\partial\eta} = \frac{-f(z)}{1 - J\,f(z)} \tag{15}$$

where z defined by equation (10), is linked by (14) and $f(z) = dF(z)/dz$ is the probability density.

Given equations (11) and (14) , the global level of demand is :

$$Q^d(P) \equiv N\,\eta^d((P) \tag{16}$$

The resulting elasticity-price of the demand is not related to size N of the population :

$$-\epsilon(P, \eta) = \frac{d\eta^d(P)}{dP}\,\frac{P}{\eta^d(P)} = \frac{-f(z)\,P}{(1 - J\,f(z))\,\eta^d(P)} \tag{17}$$

Since for a given P, equation (12) finally defines the penetration rate η as a fixed-point. For low value of β, (single fixed point), inversion of this equation gives a dimensionless *inverse demand function*:

$$P^d(\eta) = H + J\,\eta + \frac{1}{\beta}\ln\frac{1-\eta}{\eta} \tag{18}$$

4 The distribution of optimal asymptotic prices for a monopolist: first investigations

On the supply side, we consider a monopolist facing heterogeneous customers in a risky situation where this seller has perfect knowledge of the functional form of the agents' surplus functions and their related maximisation behaviour (1). He also knows the size of the population and the statistical (logistic) distribution of the idiosyncratic part of the reservation prices (H_i). But, in the market process, the monopolist cannot observe any of these *individual* reservation prices. He observes only the result of the individual choices (to buy or not to buy). In the case of "global" externality, the interactions are the same for all customers, and all agents are fully interconnected with each others (equation 4). Then, as just seen with equation (11), the greater N is, the closer to $J\,\eta$ is the social influence on each individual decision. Because the monopolist observes the number of buyer, he also know η (the fraction of customers) in the whole population. As a consequence, in the case of constant marginal cost the monopolist can maximise indifferently the total expected profit or the per unit expected profit, with an expected demand given by equation (11). Analytical results presented below summarize Nadal *et al.* [16] [2].

4.1 Profit maximisation

Let $N\eta^s$ be the number of good to be put on the market by the monopolist, and C a constant per unit cost of production. The expected profit if he sells the good at price P is then $N\Pi(P, \eta^s)$ with

$$\Pi(P, \eta^s) \equiv P\,\eta^s \;-\; C\,\eta^s, \qquad\qquad \eta^s \leq \eta^d(P) \tag{19}$$

Equation (16) defines $Q^d(P) \equiv N\,\eta^d((P)$ as the expected agregate demand of consumers at price P. In situation of risk, the monopolist can build this demand index from the fixed point relationship between the penetration rate η (proportion of buyers in the whole market N) and the mathematical expectation to have a buyer by taking an agent at random in this population - equations (11),(13). Let p be the profit *per unit*:

$$p \equiv P - C \tag{20}$$

Since $P - H = (P - C) - (H - C)$, defining:

$$h \equiv H - C, \tag{21}$$

we can rewrite z in (10) and (11) as:

$$z = p \;-\; h \;-\; J\,\eta. \tag{22}$$

Hereafter we write all the equations in terms of p and h.

The problem for the monopolist is to find η^M and p^M, the values of η^s and p which maximise its profit (19). The monopolist's program is then to solve:

$$(p^M, \eta^M) \in \{\arg\max_{p,\eta^s} \Pi(p, \eta^s)/sc : \eta^s \leq \eta^d(p)\} \tag{23}$$

with : $\Pi(p, \eta^s) = p\eta^s$. Optimal conditions for the program (23) hold :

$$\eta^s + \lambda\,\frac{d\eta^d(p)}{dp} = 0 \tag{24}$$

$$p - \lambda = 0 \tag{25}$$

$$\eta^s = \eta^d(p) \tag{26}$$

From (25) we have $\lambda = p$. Then, optimal conditions (24),(25) can be reduced at the following single condition:

[2] available at: http://www-eco.enst-bretagne.fr/~phan/papers/npgweiha2003.pdf
the authors aknowledge Mirta B. Gordon for her significant contribution to this results

$$\eta^s + p\,\frac{d\eta^d(p)}{dp} = 0 \tag{27}$$

which gives $\eta^s(p)$, the supply function as a function of p,

$$\eta^s(p) \equiv -p\,\frac{d\eta^d(p)}{dp} \tag{28}$$

At the optimum, given the the exclusion condition (26), p^M is (one of the) solution(s) of

$$\eta^s(p) = \eta^d(p) \tag{29}$$

Finally, we obtain p_M and η_M at the intersection between supply and demand :

$$\eta^M = \eta^d(p^M) = \eta^s(p^M). \tag{30}$$

Taking in supply function (28) the derivative as given implicitly by (15), equilibrium condition (30) lead, after some some manipulations, to the classical Lener index of monopolist's power:

$$\frac{P_M - C}{P_M} = \frac{(1 - Jf(z))\,\eta_M}{f(z)\,P_M} = \frac{1}{\epsilon(P_M, \eta_M)}, \tag{31}$$

4.2 Phase transition in the monopolist's strategy

Figures (6) & (7) summarize results on long run optimal regimes for monopoly, including possibility of transition between regimes for some critical values from from Nadal *et al.* [16].
In the special case where $h < 0$ (the majority of the population is not willing to buy) two differents regimes may exist, for J sufficient high. As βJ increases, the optimal monopolist's strategy shifts abruptly from a regime with high price and a small fraction of buyers to a regime of low price with a large fraction of buyers.

5 Conclusion

In this paper, we assumed a positive (additive) effect of the social influence on the willingness to pay. Heterogeneous agents have a fixed idiosyncratic part in this willingness to pay, unobservable by the monopolist. Numerous models of social interaction often used in economics have an additive random (logistic) part in their willingness to pay [8, 5, 3, 11, 19], which corresponds in physics to a case of 'annealed' disorder. With fixed agent's heterogeneous idiosyncratic characteristic, the model is equivalent to the 'Random Field Ising model', belonging to the class of 'quenched' disordered models widely studied in statistical physics. These two classes of models generally differ, except for the equilibrium properties in the special case of homogeneous interactions with global interactions.

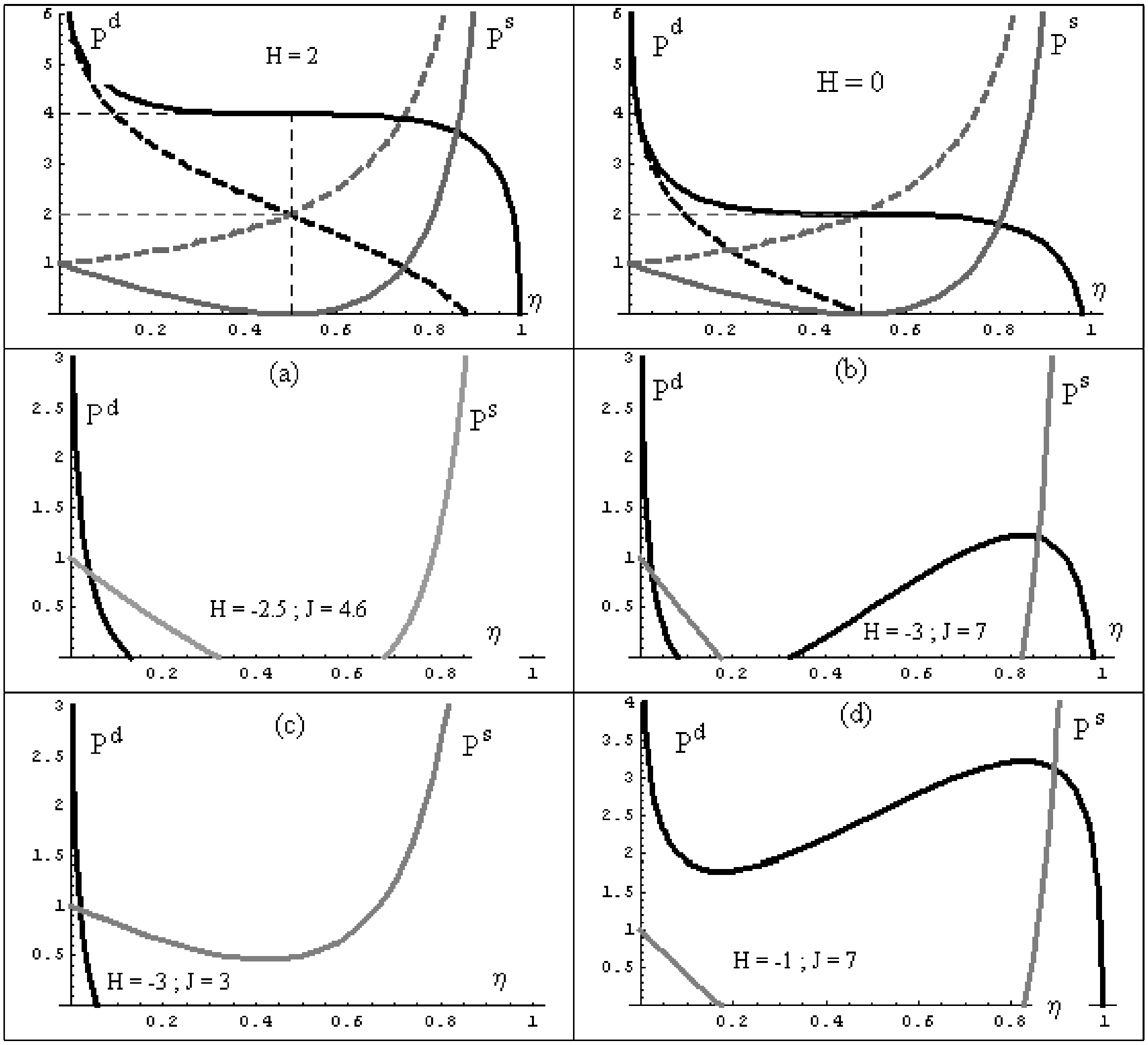


Figure 6: Inverse supply and demand curves $P^s(\eta)$ and $P^d(\eta)$, for different values of H and J ($\beta = 1$). As $C = 0$, we have: $H = h$. The first and second graphic show the difference between absence of externality ($J = 0$, dashed lines) and strong externality case, ($J = 4$, solid lines). The case $h = 2$ corresponds to a strong positive average of the population's IWP, and the population is neutral for $h = 0$. Negative idiosyncrasic average willingness to pay ($h < 0$), in the following four graphics means that only few consumer are interested for this commodity whithout externality. These graphics, labelled (a) to (d), corresponding to the points labelled (a) to (d) in the phase diagram (figure 7). The social effects and the inhomogeneity allow us to find equilibrium prices (the intersection between the demand (black) and the supply (grey) curves). (a) corresponds to the *coexistence region* between two local market equilibria in (figure 7), but one of them, corresponding to a negative price solution is not relevant (not shown). (b) corresponds to the *coexistence region*; in this case, the high η market equilibrium is the optimal one. (c) lies in the region with only one market equilibrium, with few buyers (small η). (d) corresponds to a large social effect, the single market equilibrium has large η and a high price. Source: Nadal *et al.* [16].

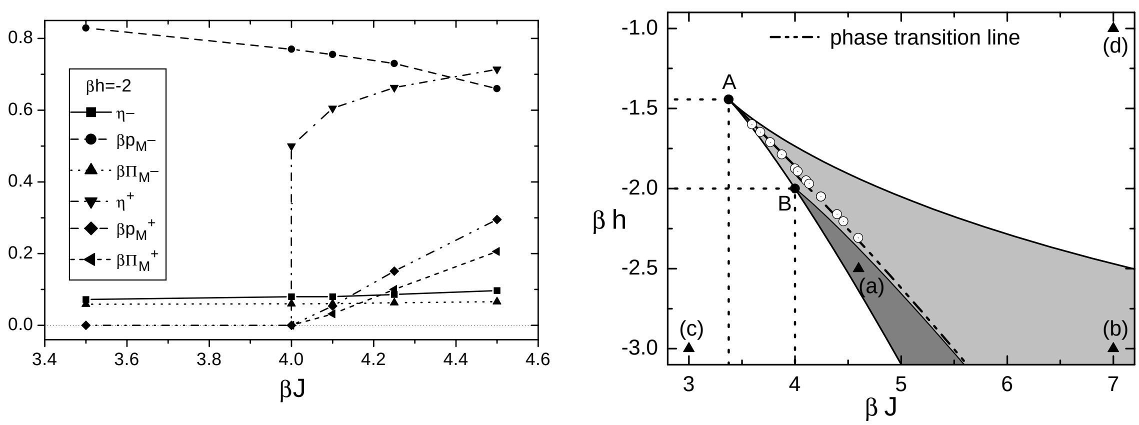


Figure 7: (A) Fraction of buyers η, optimal price βp_M and monopolist profit Π_M, as a function of the social influence, for $\beta h = -2$. The superscripts $-$ and $+$ refer to the two solutions of equations (30) that are relative maxima.
(B)Phase diagram in the plane $\{\beta J, \beta h\}$: the grey region represents the domain in the parameter space where coexist two maxima of the monopolist's profit, a global one (the optimal solution) and a local one. Inside this domain, as βJ and/or βh increase, there is a (first order) transition where the monopolist's optimum jumps from a high price, low penetration rate solution $\eta = \eta_-$ to one with low price, large $\eta = \eta_+$. The circles on the transition line have been obtained numerically, the smooth curves are obtained analytically (see the Appendix and [16] for details). The points (a) to (d) correspond to the inverse supply and demand curves represented in figure 6. In the white region, for $\beta J < 27/8$, the fraction of buyers, η, increases continuously from 0 to 1 as βh increases from $-\infty$ to $+\infty$ (c-d). At the singular point A, ($\beta J = 27/8$, $\beta h = -3/4 - \log(2)$), $\eta_+ = \eta_- = 1/3$. At point B ($\beta J = 4$, $\beta h = -2$), the local maximum with η large appears with a null profit and $\eta_+ = 1/2$. In the dark-grey region below B, this local maximum exists with a negative profit, being thus non viable for the monopolist (a). Source: Nadal *et al.* [16].

In this special situation, which corresponds to *mean-field approximation* in physics, the static (long run) optimal solution is the same in both models [16, 19].

In the random field case, studied here, since the distribution of agents over the network is random, the resulting organisation is complex. ACE Computational Laboratories Moduleco provides a useful and friendly framework for to model, investigate and understand the dynamics of such complex adaptive systems. The strategy followed here is to use ACE as a *complement* to the mathematical theorising, rather than a complete substitute [2].

In the model presented here, the optimal asymptotic monopolist price is known analytically in two polar cases: without externality or with global externality. Analytical results may be possible for the homogeneous regular case, but in more general cases (including the so-called " small world", characterised by both highly local and regular connections and some long range, disordered connections), numerical (statistical) results are often the only possible way. However, present results allow us to observe numerous complex dynamics on the demand side, such as hysteresis, avalanches or Sethna's inner loop hysteresis. As a result, the seller's problem is generally non trivial, even in the case of risk, where the seller knows all the parameters of the customer's program and the initial distribution of the idiosyncratic parameters.

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